The New World of Neutrino Physics

Boris Kayser
European School of HEP
June 29, 2006
What We Have Learned
The (Mass)$^2$ Spectrum

\[
\begin{align*}
\nu_3 & \quad \Delta m^2_{\text{atm}} \quad \text{or} \quad \Delta m^2_{\text{sol}} \\
\nu_2 \quad & \quad \Delta m^2_{\text{sol}} \\
\nu_1 & \quad \Delta m^2_{\text{atm}} \\
\end{align*}
\]

\[
\Delta m^2_{\text{sol}} \approx 8 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{\text{atm}} \approx 2.7 \times 10^{-3} \text{ eV}^2
\]

Are there more mass eigenstates, as LSND suggests?
Leptonic Mixing

This has the consequence that —

$$|\nu_i> = \sum_{\alpha} U_{\alpha i} |\nu_\alpha>.$$  

Flavor-\(\alpha\) fraction of \(\nu_i = |U_{\alpha i}|^2\).

When a \(\nu_i\) interacts and produces a charged lepton, the probability that this charged lepton will be of flavor \(\alpha\) is \(|U_{\alpha i}|^2\).
The spectrum, showing its approximate flavor content, is

\[
\nu_3 \quad \Delta m^2_{\text{atm}} \quad \sin^2 \theta_{13} \quad \nu_2 \quad \Delta m^2_{\text{sol}} \quad \nu_1
\]

\[
\Delta m^2_{\text{atm}} \quad \Delta m^2_{\text{sol}}
\]

or

\[
\nu_3 \quad \sin^2 \theta_{13} \quad \nu_2 \quad \Delta m^2_{\text{atm}} \quad \nu_1
\]

Normal

Inverted

\[\nu_e [ | U_{ei} |^2] \quad \nu_\mu [ | U_{\mu i} |^2] \quad \nu_\tau [ | U_{\tau i} |^2]\]
Bounded by reactor exps. with $L \sim 1$ km

From max. atm. mixing, $\nu_3 \approx \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$

\[
\Delta m^2_{\text{atm}} \begin{cases} 
\text{From } \nu_\mu \text{(Up) oscillate} \\
\text{but } \nu_\mu \text{(Down) don’t} 
\end{cases}
\]

In LMA–MSW, $P_{\text{sol}}(\nu_e \rightarrow \nu_e)$

\[
= \nu_e \text{ fraction of } \nu_2
\]

\[
\Delta m^2_{\text{sol}} \quad \text{From distortion of } \nu_e(\text{solar})\text{ and } \overline{\nu}_e(\text{reactor}) \text{ spectra}
\]

\[
\begin{cases}
\text{From max. atm. mixing, } \nu_1 + \nu_2 \\
\text{includes } (\nu_\mu - \nu_\tau) / \sqrt{2}
\end{cases}
\]
The Mixing Matrix

\[ U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ c_{ij} \equiv \cos \theta_{ij} \]
\[ s_{ij} \equiv \sin \theta_{ij} \]

\[ \theta_{12} \approx \theta_{\text{sol}} \approx 34^\circ, \quad \theta_{23} \approx \theta_{\text{atm}} \approx 37-53^\circ, \quad \theta_{13} \lesssim 10^\circ \]

\[ \delta \text{ would lead to } P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta). \]

But note the crucial role of \( s_{13} \equiv \sin \theta_{13}. \)
The Majorana CP Phases

The phase $\alpha_i$ is associated with neutrino mass eigenstate $\nu_i$:

$$U_{\alpha i} = U_{\alpha i}^0 \exp(i\alpha_i/2)$$ for all flavors $\alpha$.

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^* \exp(-im_i^2L/2E) U_{\beta i}$$

is insensitive to the Majorana phases $\alpha_i$.

Only the phase $\delta$ can cause CP violation in neutrino oscillation.
The Open Questions
• What is the absolute scale of neutrino mass?

• Are neutrinos their own antiparticles?

• Are there “sterile” neutrinos?

We must be alert to surprises!
• What is the pattern of mixing among the different types of neutrinos?

  What is $\theta_{13}$? Is $\theta_{23}$ maximal?

• Is the spectrum like $\equiv$ or $\equiv$?

• Do neutrinos violate the symmetry CP?

  Is $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?
• What can neutrinos and the universe tell us about one another?

• Is CP violation by neutrinos the key to understanding the matter–antimatter asymmetry of the universe?

• What physics is behind neutrino mass?
The Importance of the Questions, and How They May Be Answered
What Is the Absolute Scale of Neutrino Mass?

How far above zero is the whole pattern?

\[(\text{Mass})^2\]

\(\nu_3\)  \(\Delta m^2_{\text{atm}}\)  \(\nu_2\)  \(\nu_1\) \(\nu\) Flavor Change

\(> \Delta m^2_{\text{sol}}\)

\{ Tritium Decay, Double \(\beta\) Decay Cosmology \}
A Cosmic Connection

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{\text{atm}}} < \text{Mass}[\text{Heaviest } \nu_i]$

Cosmological Data + Cosmological Assumptions $\Rightarrow \Sigma m_i < (0.17 - 1.0) \text{ eV}$. 

If there are only 3 neutrinos,

$0.04 \text{ eV} \lesssim \text{Mass}[\text{Heaviest } \nu_i] < (0.07 - 0.4) \text{ eV}$

(Seljak, Slosar, McDonald)

Pastor
To Determine If Neutrinos Are Majorana Particles
How Can We Demonstrate That $\bar{\nu}_i = \nu_i$?

We assume neutrino interactions are correctly described by the SM. Then the interactions conserve $L$ ($\nu \rightarrow \ell^- ; \bar{\nu} \rightarrow \ell^+$).

**An Idea that Does Not Work**

[and illustrates why most ideas do not work]

Produce a $\nu_i$ via—

\[
\begin{array}{c}
\nu_i \\
\Rightarrow
\
\pi^+ \\
\Rightarrow
\
\mu^+
\end{array}
\]

**Spin**

Pion Rest Frame

Give the neutrino a Boost:

$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$

\[
\begin{array}{c}
\pi^+ \\
\Rightarrow
\
\nu_i \\
\Rightarrow
\
\mu^+
\end{array}
\]

Lab. Frame
The SM weak interaction causes—

\[ \nu_i \Rightarrow \bar{\nu}_i \Rightarrow \mu^+ \]

Target at rest

Recoil

If \( \nu_i = \bar{\nu}_i \) means that \( \nu_i(h) = \bar{\nu}_i(h) \).

our \( \nu_i \Rightarrow \) will make \( \mu^+ \) too.
Minor Technical Difficulties

\[ \beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame}) \]

\[ \Rightarrow \frac{E_\pi(\text{Lab})}{m_\pi} > \frac{E_\nu(\pi \text{ Rest Frame})}{m_\nu_i} \]

\[ \Rightarrow E_\pi(\text{Lab}) \geq 10^5 \text{ TeV} \text{ if } m_\nu_i \sim 0.05 \text{ eV} \]

Fraction of all \( \pi \) – decay \( \nu_i \) that get helicity flipped

\[ \approx \left( \frac{m_\nu_i}{E_\nu(\pi \text{ Rest Frame})} \right)^2 \sim 10^{-18} \text{ if } m_\nu_i \sim 0.05 \text{ eV} \]

Since \( L \)-violation comes only from Majorana neutrino masses, any attempt to observe it will be at the mercy of the neutrino masses.

(BK & Stodolsky)
The Idea That Can Work —
Neutrinoless Double Beta Decay $[0\nu\beta\beta]$

By avoiding competition, this process can cope with the small neutrino masses.

Observation would imply $\mathcal{L}$ and $\bar{\nu}_i = \nu_i$.  

Nucl $\rightarrow$ Nuclear Process $\rightarrow$ Nucl'

$\sum_i \nu_i W^- W^- e^- e^-$
Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

\[ (\bar{\nu})_R \rightarrow \nu_L : \text{A Majorana mass term} \]
In —

\[ \sum_i U_{ei} \overline{\nu}_i \rightarrow \text{Nuclear Process} \rightarrow \sum_i U_{ei} \nu_i \]

the $\overline{\nu}_i$ is emitted [$RH + O\{m_i/E\}LH$].

Thus, Amp [$\nu_i$ contribution] \( \propto m_i \)

\[ \text{Amp}[0\nu\beta\beta] \propto \left| \sum_i m_i U_{ei}^2 \right| \equiv m_{\beta\beta} \]
The proportionality of $0\nu\beta\beta$ to mass is no surprise.

$0\nu\beta\beta$ violates $L$. But the SM interactions conserve $L$.

The $L$ – violation in $0\nu\beta\beta$ comes from underlying **Majorana** mass terms.
Wouldn’t the dependence on neutrino mass be eliminated by a Right-Handed Current?

The SM LH current does not violate L. An identical current, but of opposite handedness, wouldn’t violate L either.

We still need the L-violating Majorana neutrino mass to make this process occur.
With a RH current at one vertex,

$$\text{Amp}[0\nu\beta\beta] \propto (\nu \text{ mass})^2.$$ 

Contributions with a RH current at one vertex are not likely to be significant.

\{BK, Petcov, Rosen
\{Enqvist, Maalampi, Mursula\}
How Large is $m_{\beta\beta}$?

How sensitive need an experiment be?

Suppose there are only 3 neutrino mass eigenstates. (More might help.)

Then the spectrum looks like —

\[
\begin{align*}
\text{sol} < \nu_2 & \quad \text{atm} \quad \nu_3 \\
\text{sol} < \nu_1 & \quad \nu_3
\end{align*}
\]

or

\[
\begin{align*}
\text{sol} < \nu_2 & \quad \text{atm} \quad \nu_1 \\
\text{sol} < \nu_3
\end{align*}
\]
\[ m_{\beta\beta} \equiv | \sum m_i U_{ei}^2 | \]

\[
U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

The e (top) row of \( U \) reads —

\((U_{e1}, U_{e2}, U_{e3}) = (c_{12} c_{13} e^{i\alpha_1/2}, s_{12} c_{13} e^{i\alpha_2/2}, s_{13} e^{-i\delta})\)

\( \theta_{12} \approx \theta_{\odot} \approx 34^\circ, \) but \( s_{13}^2 < 0.032 \)
If the spectrum looks like—

\[
m_{\beta\beta} \cong m_0 \left[ 1 - \sin^2 2\theta_\odot \sin^2 \left( \frac{\alpha_2 - \alpha_1}{2} \right) \right]^{1/2}.
\]

then—

\[
m_0 \cos 2\theta_\odot \leq m_{\beta\beta} \leq m_0
\]

At 90% CL,

\[
m_0 > 40 \text{ meV (SuperK)}; \cos 2\theta_\odot > 0.28 \text{ (SNO)},
\]
so

\[
m_{\beta\beta} > 11 \text{ meV}.
\]
If the spectrum looks like

\[ 0 < m_{\beta\beta} < \text{Present Bound } [(0.3 – 1.0) \text{ eV}] \]

(Petcov et al.)

**Analyses of** \( m_{\beta\beta} \) **vs. Neutrino Parameters**
Barger, Bilenky, Farzan, Giunti, Glashow, Grimus, BK, Kim, Klapdor-Kleingrothaus, Langacker, Marfatia, Monteno, Murayama, Pascoli, Päs, Peña-Garay, Peres, Petcov, Rodejohann, Smirnov, Vissani, Whisnant, Wolfenstein,

Review of \( \beta\beta \) Decay: Elliott & Vogel

Evidence for \( 0\nu\beta\beta \) with \( m_{\beta\beta} = (0.05 – 0.84) \text{ eV} \)

Klapdor-Kleingrothaus
Are There Sterile Neutrinos?

*Rapid* neutrino oscillation reported by **LSND** —

\[ \Delta m^2_{atm} = 2.7 \times 10^{-3} \text{ eV}^2 \]
\[ \Delta m^2_{sol} = 8 \times 10^{-5} \text{ eV}^2 \]

At least 4 mass eigenstates, hence at least 4 flavors.

Measured \( \Gamma(Z \rightarrow \nu \bar{\nu}) \) only 3 different *active* neutrinos.

At least 1 *sterile* neutrino.
Is the so-far unconfirmed oscillation reported by LSND genuine?

MiniBooNE aims to definitively answer this question.
What Is the Pattern of Mixing?

How large is the small mixing angle $\theta_{13}$?

We know only that $\sin^2\theta_{13} < 0.032$ (at 2$\sigma$).

The theoretical prediction of $\theta_{13}$ is not sharp:

[Diagram showing number of models vs $\sin^2\theta_{13}$]
The Central Role of $\theta_{13}$

Both CP violation and our ability to tell whether the spectrum is normal or inverted depend on $\theta_{13}$.

If $\sin^2\theta_{13} > (0.0025 - 0.0050)$, we can study both of these issues with intense but conventional $\nu$ and $\bar{\nu}$ beams.

Determining $\theta_{13}$ is an important stepping-stone.
How $\theta_{13}$ May Be Measured

$\sin^2 \theta_{13} = \left| U_{e3} \right|^2$ is the small $\nu_e$ piece of $\nu_3$.

$\nu_3$ is at one end of $\Delta m^2_{\text{atm}}$.

$\therefore$ We need an experiment with L/E sensitive to $\Delta m^2_{\text{atm}}$ ($L/E \sim 500 \text{ km/GeV}$), and involving $\nu_e$. 
Complementary Approaches

Reactor Experiments

Reactor $\overline{\nu}_e$ disappearance while traveling $L \sim 1.5$ km. This process depends on $\theta_{13}$ alone:

$$P(\overline{\nu}_e \text{ Disappearance}) =$$

$$= \sin^2 2\theta_{13} \sin^2 [1.27 \Delta m^2_{\text{atm}} (\text{eV}^2)L(\text{km})/E(\text{GeV})]$$
Accelerator Experiments

Accelerator $\nu_\mu \to \nu_e$ while traveling $L > $ Several hundred km. This process depends on $\theta_{13}$, $\theta_{23}$, on whether the spectrum is normal or inverted, and on whether CP is violated through the phase $\delta$. 
Neglecting matter effects (to keep the formula from getting too complicated):

The accelerator long-baseline ($\overline{\nu}_e$) appearance experiment measures —

\[
P[\nu_\mu \rightarrow \nu_e] \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \Delta_{31} \\
+ \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \pm \delta) \\
+ \sin^2 2\theta_{12} \cos^2 \theta_{23} \cos^2 \theta_{13} \sin^2 \Delta_{21}
\]

\[
\Delta_{ij} \equiv \Delta m_{ij}^2 L / 4E
\]

The plus (minus) sign is for neutrinos (antineutrinos).
What is the atmospheric mixing angle $\theta_{23}$?

$$P[\nu_\mu \rightarrow \text{Not } \nu_\mu] \approx \sin^2 2\theta_{23} \sin^2 \Delta_{atm}$$

Here $\Delta_{atm}$ lies between the (very nearly equal) $\Delta_{31}$ and $\Delta_{32}$.

This measurement determines $\sin^2 2\theta_{23}$, but if $\theta_{23} \neq 45^\circ$, there are two solutions for $\theta_{23}$:

$$\theta_{23} \text{ and } 90^\circ - \theta_{23}.$$  

A reactor experiment may be able to resolve this ambiguity.
Assumes $\sin^2 2\theta_{23} = 0.95 \pm 0.01$

Sensitive to $\sin^2 2\theta_{13} = 0.01$

(McConnel, Shaevitz)
The Mass Spectrum: $\equiv$ or $\equiv$ ?

Generically, grand unified models (GUTS) favor —

—

GUTS relate the Leptons to the Quarks.

— is un-quark-like, and would probably involve a lepton symmetry with no quark analogue.
How To Determine If The Spectrum Is Normal Or Inverted

Exploit the fact that, in matter,

\[ \nu_e (\nu_e) \]

raises the effective mass of \( \nu_e \), and lowers that of \( \bar{\nu}_e \).

This changes both the spectrum and the mixing angles.
Matter effects grow with energy $E$.

At $E \sim 1$ GeV, matter effects $\Rightarrow$

\[
\sin^2 2\tilde{\theta}_M \equiv \sin^2 2\theta_{13} \left[ 1 \, \leftarrow \, S \frac{E}{6 \text{ GeV}} \right] .
\]

Sign[$m^2(\mu) - m^2(e)$]

At oscillation maximum,

\[
\begin{align*}
P(\nu_\mu \rightarrow \nu_e) & > 1 \; ; \; \text{ } \\
P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) & < 1 \; ; \; \text{ } \\
\end{align*}
\]

Note fake CP violation.

In addition,

\[
\begin{align*}
P_{\text{Hi } E}(\nu_\mu \rightarrow \nu_e) & > 1 \; ; \; \text{ } \\
P_{\text{Lo } E}(\nu_\mu \rightarrow \nu_e) & < 1 \; ; \; \text{ } \\
\end{align*}
\]

\begin{center}
( Mena, Minakata, Nunokawa, Parke )
\end{center}