Flavour Dynamics & CP in the SM*: A Crucial Past -- and an Essential Future!

Ikaros Bigi, Notre Dame du Lac
Presentation of my lecture series guided by two general considerations

- While the case for New Physics at ~ TeV scale is as strong as ever, we cannot count on NP having a massive impact on B decays.

- I will emphasize general principles for designing strategies over specific & detailed examples

The central goal for this school as for any -- we want you to do your own thinking rather than `out-source' it!

➤ raise/ask questions !!!

Give feedback -- I think I am able to learn from mistakes
Outline of Lectures

I. Flavour Dynamics in the Second Millenium (→ 1999)
Basics of flavour dynamics & CP; CKM theory; $K^0$ & $B^0$ oscillations; the SM `Paradigm of large CP in B Decays`

II. Flavour Dynamics 2000 - 2006
Verifying `Paradigm of large CP in B Decays'; praising EPR correl. & hadronization; Heavy Quark Theory; extract. CKM param.; triangle fits

III. Probing the Flavour Paradigm of the Emerging New Standard Model
Indirect searches for NP; `King Kong' scenarios (EDM's, charm, $\tau$) vs. precision probes (B); the case for Super-B in the HEP landscape
Memento $\Delta S \neq 0$ dynamics:

- $\tau-\theta$ puzzle $\Rightarrow R!$
- production $\gg$ decay $\Rightarrow$ families!
- no $\Delta F \neq 0$ NC $\Rightarrow$ charm!
- $K_L \rightarrow \pi\pi$ $\Rightarrow$ CP, top!

All New Physics at that time

-- yet now pillars of the SM!
A famous coach once declared: “Winning is not the greatest thing -- it is the only thing!”

The `SM *' =

\[ \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1) + \text{CKM} + \text{PMNS} \]

general considerations
renormalizability + Adler anomaly + data

the `only' thing
not even the greatest thing

an accidental miracle
(P.1) QCD -- the `Only' Thing

- **chiral symmetry** ($\pi$ Goldstone bosons, soft $\pi$ theorems, etc.)
  - need **vector** couplings for gluons

- **$R(e^+e^- \rightarrow \text{had.}, \pi^0 \rightarrow \gamma\gamma$, etc. etc.**
  - need **three colours**

- **unbroken symmetry:** local gauge theory only known way to couple to $m=0, j=1$ fields in Lorentz invariant way: $4 \neq 2$!

- confinement → asymptotic freedom
  - non-abelian gauge theory

☞ **QCD -- unique choice among local quantum field theories**
SU(2)_L \times U(1) -- not even the Greatest Thing

Strong points

renormalizibility (+unitarity) severely restrict possible theories (problem of mass -- later)

☺ single SU(2)_L \rightarrow weak universality due to self coupling of gauge bosons

☺ predicted
- existence of NC parametrized by 1 parameter \( \sin \theta_W \)
- \( M_W, M_Z \)

☺ most remarkable: combines
- QED -- pure V coupling (P \( \sqrt{\ )} \) with \( m_\gamma = 0 \)
- with weak interactions -- V-A CC coupling (P maximal)
  & V,A NC coupling \( M_Z > M_W \neq 0 \)

☺ can generate masses for bosons & fermions through single Higgs
`quantum anomaly':

Classical conservation law vitiated due to quantum correction

\[ \partial_\mu J^5_\mu \neq 0 \text{ even for } m_f = 0 \]

- Destroys renormalizibility
- Can be neutralized (not solved) within SM by
  \[ \Sigma_f Q_f = 0 , f = \text{fermions within given family} \]
  lepton-quark connection
Theoretical Deficiencies

With all these amazing successes -- what is the fuss, why not be happy?

- SU(2)$_L \times$U(1) -- partial unification only
- HBEGHK mechanism:
  - only `engineering' solution -- at least till Higgs is found
  - scalar couplings `unnatural' (quadratic mass renormal. !)
  - justification for LHC & motivation for ILC
- maximal $\mathcal{P}$ (for CC) `par ordre du mufti'
- $m_\nu = 0$ (up to Majorana) `par ordre du mufti'
- charge quantization

... and then the whole issue of family replication!
3 families

\[
\begin{array}{ccc}
\nu_e & u & \nu_\mu \\
e & d & \mu \\
\nu_\tau & \tau & b
\end{array}
\]

- Why > 1 family? Why 3? ?? M theory ??
- Is \(N_{\text{fam}}\) a fundamental quantity?

Evidence for us being `dense'/`blind' is even stronger!
\[ m_t \sim 175 \text{ GeV} \]
\[ m_b \sim 4.6 \text{ GeV} \]
\[ m_c \sim 1.2 \text{ GeV} \]
\[ m_s \sim 0.1 \text{ GeV} \]
\[ m_u \sim \text{a few MeV} \]
\[ m_d \sim \text{a few more MeV} \]
\[ \Lambda_{\text{QCD}} \]
\[ m_\tau \sim 1.7 \text{ GeV} \]
\[ m_\mu \sim 0.1 \text{ GeV} \]
\[ m_e \sim 0.5 \text{ MeV} \]
\[ \Delta m^2 \sim O(10^{-3} \text{ eV}^2), O(10^{-4} \text{ eV}^2) \]
\[ \nu \text{ oscillations} \]

\[ m_\nu(\tau) < 18.2 \text{ MeV} \]
\[ m_\nu(\mu) < 0.19 \text{ MeV} \]
\[ m_\nu(e) < 3 \text{ eV} \]

+ structure of CKM matrix (later)!
Basics of P, C, CP & T

Definitions

- **Parity P** (or mirror reflection)
  
  \[
  x \rightarrow -x, \quad p \rightarrow -p \\
  l = x \times p \rightarrow l \\
  p_1 \cdot p_2 \rightarrow p_1 \cdot p_2 \\
  l \cdot p \rightarrow -l \cdot p \\
  \dagger \rightarrow \dagger 
  \]

  Transformation operator \( P \) linear & unitary

- **Charge Conjugation C**
  
  Particle \( \rightarrow \) anti-particle

  Positive charge \( \rightarrow \) negative charge

  Transformation operator \( C \) linear & unitary
time reversal $T$ (or `reversal of motion')

$x \rightarrow x, \, t \rightarrow -t$

$p \rightarrow -p, \, l \rightarrow -l$

$l \cdot p \rightarrow l \cdot p$

$l \cdot (p_1 \times p_2) \rightarrow -l \cdot (p_1 \times p_2)$

transformation operator $T$ (anti-linear & unitary)

anti-linear: $T(\alpha |a> + \beta |b>) = \alpha^* T |a> + \beta^* T |b>$

why? Invariance of CCR: $[X,P] = i1$

$- [X,P] = T[X,P] T^* = T \cdot i1 \cdot T^* = -i1$

$CP = T$

$CP \iff$ complex phase
Kramers' degeneracy

T antiunitary: $T^2 = +1$ or -1

if $T^2 = -1 \Rightarrow$ (at least) doubly degenerate

$T^2 = -1$: 'fermionic' d.o.f. without ref.

$T^2 = +1$: 'bosonic' d.o.f. to spin!

practical application:

odd vs. even number electron systems in external electric field
(1.2) Macroscopic (= `Arrow of Time`) $T$

classical mechanics: billiard balls

- $A \rightarrow B$ and $B \rightarrow A$ equally possible
- $T(A \rightarrow B) = T(B \rightarrow A)$
possible & ordinary

possible -- yet unlikely
needs fine-tuning of initial condit.

possible & very ordinary

practically impossible
2nd example: $n \rightarrow p e \nu$ vs. $p e \nu \rightarrow n$

3rd example: Quantum Mechanics

Daily experiences of $T$ represent asymmetries in macroscopic initial conditions irrespective of $T$ invariant microscopic equations of motions!

Yet microscopic $T$ has been observed.
(1.3) **Caveat on signatures of $T$**

- **Parity $P$:** $\langle f | l \cdot p | i \rangle = \langle f | P^*P(l \cdot p)P^*P | i \rangle = \langle f | P^*P(l \cdot p)P^* | i \rangle$,
  
  $$\langle f | P^*P(l \cdot p)P^* | i \rangle = - \langle f | l \cdot p | i \rangle$$

  $\Rightarrow$ **$P$-odd correlation/moment $\neq 0 \Rightarrow P$**

- **But:** **$T$-odd correlation/moment $\neq 0$ does not necessarily imply $T$!**

- $\forall$ for $A \rightarrow B+C$ one does /cannot implement $B+C \rightarrow A$

- **Final state interactions (FSI):**

  $$T(\exp(\int dtH))T^* = (\exp - i \int dtH) \neq (\exp i \int dtH)$$
  even if $[T,H] = 0$, since $T$ antilinear!

  $\Rightarrow$ **FSI can fake $T$**
Very special role of CP & CP

CP vs. P

Wick ND, Wightman, Wigner (1952): the "... disturbing possibility ..." that CP \( \sqrt{ } \), yet P and C is "remote at the moment"!

discovery of P in '57 a great shock, yet theorists quickly recovered "politics and P"

\[
\begin{align*}
\pi^- & \rightarrow e^-_L \nu \\
\pi^+ & \rightarrow e^+_R \nu
\end{align*}
\]

"L" = f ("-")

CP: \((\pi^- \rightarrow e^-_L \nu) \Rightarrow (\pi^+ \rightarrow e^+_R \nu)\)

If CP \( \sqrt{ } \) ⇒ "L" pure convention!

"the thumb is left on the right hand!"
- \( \mathcal{CP} \Rightarrow \nabla \)

- \( \mathcal{CP} \) required to define matter vs. antimatter, L vs. R, + vs. - in convention independent way

- smallest observed violation of a symmetry

\[
\text{Im } M_{12} \approx 1.1 \times 10^{-8} \text{ eV} \iff \text{Im } M_{12}/m_K \approx 2.2 \times 10^{-17}
\]

- '65: Sakharov conditions for baryogenesis
  - \( \Delta N_{\text{baryon}} \neq 0 \),
  - \( \mathcal{CP} \)
  - out-of-thermal equilibrium
(2) Flavour Dynamics and the CKM Ansatz

(2.1) Pre-SM Flavour Dynamics & GIM

- striking feature in the data:
  huge suppression of FlChNC observed
  \[ \Gamma(K^+ \to \pi^+e^+\nu) / \Gamma(K^+ \to \pi^0e^+\nu) \sim 6 \times 10^{-6} \]
  \[ \Gamma(K_L \to \mu^+\mu^-) / \Gamma(K^+ \to \mu^+\nu) \sim 3 \times 10^{-9} \]

some even suggested the observed huge suppression of strangeness (flavour) changing NC implied a similar reduction for all NC

yet in Cabibbo theory

\[ J_{\ldots CC} \propto \cos \theta_C \ d_L \gamma \ldots u_L + \sin \theta_C \ s_L \gamma \ldots u_L \]
\[ \Rightarrow [J_{\ldots +}, J_{\ldots -}] \propto \ldots + \sin \theta_C \ s_L \gamma \ldots d_L \]

strangeness (flavour) changing NC!
\[\textit{GIM mechanism}: \text{use 4th quark, charm}\]

-- originally introduced to complete 2nd family --
to suppress $\Delta S \neq 0$ NC sufficiently (see below)

for most places (outside Nagoya University)

\[\text{3 quarks: u,d,s}\]

\[\text{quarks mathematical entities}\]

\[\text{typical attitude: "Nature is smarter than Shelly (Glashow) -- she can do without charm"}\]

-- till charm was observed following the `Octobre Revolution of 1974'!
(2.2) Quark Masses, CKM & CP

\[
U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}
\]

Mass eigenstates ≠ interaction eigenstates

\[
\mathcal{L}_{CC} \propto \bar{g}_W U_L^F \gamma_\mu D_L^F W_\mu, \quad \mathcal{L}_{NC}^{U[D]} \propto \bar{g}_Z \bar{U}[D]_L^F \gamma_\mu U[D]_L^F Z_\mu
\]

\[
\mathcal{L}_M \propto \bar{U}_L^F m_U U_R^F + \bar{D}_L^F m_D D_R^F \propto \bar{U}_L^F g_U^Y U_R^F \Phi_U + \bar{D}_L^F g_D^Y D_R^F \Phi_D
\]

\[
m_{\ldots} = <\Phi_{\ldots}> g_{\ldots}^Y
\]

\[
m_{\ldots}, g_{\ldots}^Y \text{ non-diagonal in general, diagonalized by unitary } j_{U,L/R}, j_{D,L/R}
\]

- EV's of \( m_{U,D} \rightarrow \text{physical masses of } U, D
- \mathcal{L}_{NC}^{U[D]} \rightarrow \bar{U}[D]_L^m \gamma_\mu U[D]_L^m Z_\mu
- \mathcal{L}_{CC} \rightarrow \bar{U}_L^m \gamma_\mu V_{CKM} D_L^m W_\mu \quad V_{CKM} = j_{U,L} j_{D,L}^*
weak neutral currents couplings unaffected on tree-level
`generalized GIM' mechanism

\[ V_{CKM} = J_{U,L} J_{D,L}^* \]

\[ V_{CKM} = J_{U,L} J_{D,L}^* \text{ nontrivial} \]
(unless high scale dynamics enforces alignment between U & D)

weak charged currents couplings affected

N families: N x N matrix that is unitary due to 2 facts
(i) \( J_{U,L/R}, J_{D,L/R} \) unitary by construction
(ii) \( L_{CC} \propto \bar{g}_W U_L^F \gamma_\mu D_L^F W_\mu \)
SM: single SU(2) group

gauge coupling \( \bar{g}_W \) of W to fermions controlled by single self-coupling of W's

`weak universality' \( |V(ud)|^2+|V(us)|^2+|V(ub)|^2=1 \) etc.
Can weak universality be violated?

Yes -- it can

- horizontal gauge interactions = FlChNC
- couple one separate SU(2)_L to each family

-- i.e. gauge group SU(2)_L \times SU(2)_L \times SU(2)_L -- while allowing those three sets of gauge bosons to mix; the mass eigenstates of these W^{1}_L can be such that the lightest couple to all families with universal strength

- weak universality only approximate
- induce FlChNC ... & EDM's
N x N unitary matrix

- N (weak) universality relations
  \[ \sum_j |V(ij)|^2 = 1, \quad i=1,\ldots, N \]
  important -- yet insensitive to complex phases
  ➔ tells us nothing directly about CP

- N^2 - N orthogonality relation
  \[ \sum_j V^*(ij)V(jk) = 0, \quad i \neq k \]
  very sensitive to complex phases
  ➔ tells us directly about CP

Caveat:

- the phase of a fermion field is not always an observable!
Observable parameters of $N \times N$ unitary matrix

- $N \times N$ complex matrix: $2N^2$ real parameters
- unitary reduces it to $N^2$ independent real parameters
- phases of quark fields can be rotated freely
  - $2N-1$ phases can be removed (1 overall phase irrelevant)
  - $(N-1)^2$ independent physical parameters
- $N \times N$ orthogonal matrix: $N_{\text{angles}} = \frac{1}{2} N(N-1)$
  - $N \times N$ unitary matrix: $N_{\text{physical phases}} = \frac{1}{2}(N-1)(N-2)$
- $N=2$: 1 angle -- Cabibbo angle -- & 0 phases
- $N=3$: 3 angles & 1 phase
- $N=4$: 6 angles & 3 phases
N=2 case:

- **2 weak universality relations:**
  \[ |V(ud)|^2 + |V(us)|^2 = 1 \]
  \[ |V(cd)|^2 + |V(cs)|^2 = 1 \]

- **2 orthogonality relations:**
  \[ V(ud)^* V(us) + V(cd)^* V(cs) = 0 \]
  \[ V(us)^* V(ud) + V(cs)^* V(cd) = 0 \]
  ➞ no relative phase
  ➞ no CP with 2 families!
N=3 case:

- **3 weak universality relations:**
  \[ |V(ud)|^2 + |V(us)|^2 + |V(ub)|^2 = 1 \]
  \[ |V(cd)|^2 + |V(cs)|^2 + |V(cb)|^2 = 1 \]
  \[ |V(td)|^2 + |V(ts)|^2 + |V(tb)|^2 = 1 \]

- **6 orthogonality relations**
  \[ \sum_{j=1}^{3} V^*(ij)V(jk) = 0, \quad i \neq k \]

  ➡️ triangle relations in the complex plane
  ➫ 6 triangles have **equal area** ← single complex phase!
  area( every triangle) = 1/2 \( J \)

  **Jarlskog** variable \( J = \text{Im}V(ud)V(cs)V^*(us)V^*(cd) \)
  if \( J = 0 \) ⇒ no \( CP \)

  ➫ orientation of triangles does **not** matter

change in phase convention!
if any pair of up- or down-type quarks were mass degenerate, then any linear combination of those two is a mass eigenstate as well, and one can remove their `CKM' parameters

up- & down-type quarks have to possess different masses to allow for CP with 3 families

Compact representation:

\[ iC = [m_U m_U^*, m_D m_D^*] \]

\[ \det C = -2J(m_t^2-m_c^2)(m_c^2-m_u^2)(m_u^2-m_t^2)(m_b^2-m_s^2)(m_s^2-m_d^2)(m_d^2-m_b^2) \]

need \( \det C \neq 0 \) for CP

CKM implementation of CP irrespective of mass generation

with SM mass generation & 1 VEV CP in Yukawa coupling, i.e. hard CP!
maximal CP?

V_{CKM} =

<table>
<thead>
<tr>
<th>V(ud)</th>
<th>V(us)</th>
<th>V(ub)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V(cd)</td>
<td>V(cs)</td>
<td>V(cb)</td>
</tr>
<tr>
<td>V(td)</td>
<td>V(ts)</td>
<td>V(tb)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
&c_{12}c_{13} & & s_{12}c_{13} & & s_{13}e^{-i\delta} \\
&-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{-i\delta} & & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{-i\delta} & & s_{23}c_{13} \\
&s_{12}s_{23}-c_{12}c_{23}s_{13}e^{-i\delta} & & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{-i\delta} & & c_{23}c_{13}
\end{align*}
\]

$\delta = 90^\circ$: `maximal' CP?

⚠️ change phase convention for quark fields --
phases of fermions like the `Scarlet Pimpernel'!
\[ \mathcal{R} \text{ maximal} \quad \iff \quad \mathcal{C} \text{ maximal} \]

i.e., CPT already enforces presence of \( \overline{\nu}_R \)

`no future generation`

`man without a future -- woman without a past`
In addition to > 2 family source for CP KM in their '73 paper list also non-minimal Higgs dynamics & right-handed currents

- Being at Nagoya University K&M had a `competitive edge'/`insider knowledge'! For most places outside Nagoya
  - 3 quarks: u,d,s
  - quarks mathematical entities
  - `Genius loci' of Nagoya University
  - home of the Sakata School
    ➤ quarks readily accepted as physical objects
  - home of Prof. Niu -- an expert in cosmic ray experiments with emulsions:
    in '71 Niu reported a candidate for charm seen
    ➤ 2 complete families were `known'
2.3 Preview of CKM Theory

- $V_{CKM}$ unitary as long as CC described by a single $SU(2)_L$

- $\begin{pmatrix}
    u & c \\
    d & s \\
    t & b
  \end{pmatrix}$

  expectation: intra- $\gg$ inter-family coupl.

inter-fam. $\sim V(us) = \sin\theta_c \sim |V(cb)|$

would imply $\tau(B) \sim \text{few } \times 10^{-14} \text{ sec}$

yet actually observed: $\tau(B) \sim 10^{-12} \text{ sec}$

$\Rightarrow |V(cb)| \sim \lambda^2$, $\lambda = \sin\theta_c$
\[ |V_{\text{CKM}}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \]

- the CKM matrix -- with this apparently highly non-accidental pattern -- describes successfully very diverse processes on vastly different scales (see later)

- Schlaeft ein Lied in allen Dingen,
  Die da traeumen fort und fort,
  Und die Welt hebt an zu singen,
  Findst Du nur das Zauberwort.

- There sleeps a song in all things
  That dream on and on,
  And the world will start to sing,
  If only you find the magic word.

  J. v. Eichendorff
the emerging pattern:

- $|V(us)| = \lambda$
- $\tau(B) \sim 1 \text{ psec} \implies |V(cb)| \sim O(\lambda^2)$
- $|V(ub)|/|V(cb)| \sim O(\lambda)$ -- i.e. ‘beauty prefers charm’

Wolfenstein representation

$$V_{CKM} = \begin{pmatrix}
1 - \lambda^2 & \lambda & \Lambda \lambda^3 (\rho - i\eta + \eta \lambda^2/2) \\
-\lambda & 1 - \lambda^2/2 - i\eta \Lambda^2 \lambda^4 & \Lambda \lambda^2 (1 + \eta \lambda^2) \\
\Lambda \lambda^3 (1 - \rho - i\eta) & -\Lambda \lambda^2 & 1
\end{pmatrix}$$
3 classes of 2 unitarity triangles each the sides of which have length

- $\lambda + \lambda + \lambda^5$

  - sd triangle: $V^*_{ud} V_{us} + V^*_{cd} V_{cs} + V^*_{td} V_{ts} = \delta_{sd} = 0$
  - cu triangle: $V^*_{ud} V_{cd} + V^*_{us} V_{cs} + V^*_{ub} V_{cb} = \delta_{cu} = 0$

- $\lambda^2 + \lambda^2 + \lambda^4$

  - bs triangle: $V^*_{us} V_{ub} + V^*_{cs} V_{cb} + V^*_{ts} V_{tb} = \delta_{bs} = 0$
  - tc triangle: $V^*_{td} V_{cd} + V^*_{ts} V_{cs} + V^*_{tb} V_{cb} = \delta_{ts} = 0$

- $\lambda^3 + \lambda^3 + \lambda^3$

  - bd triangle: $V^*_{ub} V_{ud} + V^*_{cb} V_{cd} + V^*_{tb} V_{td} = \delta_{bd} = 0$
  - tu triangle: $V^*_{ud} V_{td} + V^*_{us} V_{ts} + V^*_{ub} V_{tb} = \delta_{tu} = 0$

all six triangles have equal area!
last 2 triangles have all 3 sides of comparable length

⇒ all their angles are naturally large

control B transitions

one more obstacle: due to CPT CP can enter only through complex phases

⇒ to observe CP need 2 different, yet coherent amplitudes for a process

いますが best & most spectacular realization

\[ K^0 - K^0, B^0 - B^0 \] oscillations
`strange' particles,

\textit{since production rate >> decay rate}

\textit{new QN S s.t.}

- \textbf{strong & electromag. forces} $\Delta S=0$
- \textbf{only weak forces} $\Delta S \neq 0$

\[ K^+ \quad \bar{K}^0 \]
\[ K^0 \quad K^- \]

\[ K^0 \neq \bar{K}^0 \]

\textit{independent verification?}
Strong & e.-m. forces

\[ \Delta S = 0 \]

\[ S = -1 \quad \bar{K}^0 \]

\[ M(\bar{K}^0) = M(K^0) \quad \Gamma(\bar{K}^0) = 0 = \Gamma(K^0) \]

+ weak forces

\[ \Delta S \neq 0 \]

\[ S = +1 \quad K^0 \]

\[ K_L \propto p|K^0\rangle + q|\bar{K}^0\rangle, \quad K_S \propto p|K^0\rangle - q|\bar{K}^0\rangle \]

\[ M(K_L) > M(K_S), \quad \Gamma(K_L) \neq \Gamma(K_S) \neq 0 \]

state mixing due to SupPosPrinc of QM

40
$\text{CP} \quad \rightarrow \quad \text{mass eigenstates} = \text{CP eigenstates}

\rightarrow \text{ with CP}|K^0\rangle = |\bar{K}^0\rangle:

|K_+\rangle = (|K^0\rangle + |\bar{K}^0\rangle)/2^{1/2}, |K_-\rangle = (|K^0\rangle - |\bar{K}^0\rangle)/2^{1/2}

|K^0\rangle = (|K_+\rangle + |K_-\rangle)/2^{1/2}, |\bar{K}^0\rangle = (|K_+\rangle - |K_-\rangle)/2^{1/2}

|K^0; t\rangle = (|K_+ ; t\rangle + |K_- ; t\rangle)/2^{1/2} =

(e^{-iM(+) t}e^{-\Gamma(+) t/2}|K_+ ; t=0\rangle + e^{-iM(-) t}e^{-\Gamma(-) t/2}|K_- ; t=0\rangle)/2^{1/2} =

(e^{-iM(+) t}e^{-\Gamma(+) t/2} (|K^0\rangle + |\bar{K}^0\rangle) + e^{-iM(-) t}e^{-\Gamma(-) t/2} (|K^0\rangle - |\bar{K}^0\rangle))/2 =

(e^{-iM(+) t}e^{-\Gamma(+) t/2} + e^{-iM(-) t}e^{-\Gamma(-) t/2})|K^0\rangle + (e^{-iM(+) t}e^{-\Gamma(+) t/2} - e^{-iM(-) t}e^{-\Gamma(-) t/2})|\bar{K}^0\rangle

“spontaneous regeneration”
2 classes of decays

- **flavour specific**
  \[ \overline{K}^0 \rightarrow l^-\pi^+, l^+\pi^- \text{ vs. } K^0 \rightarrow l^+\pi^-, l^-\pi^+ \]

Homework #1: calculate these curves as \( f(\Delta M_K, \Delta \Gamma_K) \)

- **flavour non-specific**
  \[ K_+ \rightarrow 2\pi, 3\pi \text{ vs. } K_- \rightarrow 3\pi, 2\pi \]
  Due to kinematical `accident' \( 3M(\pi) \leq M(K) < 4M(\pi) \)
  \[ \Gamma(K_+) \approx 600 \Gamma(K_-) \]
  \[ \Rightarrow K_+ = K_S, K_- = K_L \]
Prediction of two neutral K mesons with vastly different lifetimes & tiny mass difference spectacularly validated. '64 an excellent year for HEP:

- Higgs mechanism for spontan. realizat. of a symmetry first developed
- Quark model (& first elements of current algebra) first suggested
- charm quark introduced
- SU(6) symmetry proposed
- first e^+e^- storage ring at Frascati
- Ω^- baryon found at Brookhaven

Shock of '64:

\[ K_\pi \rightarrow 3\pi \text{ and } 2\pi, \text{ albeit latter with tiny BR } \sim 0.0023! \]
Attempts at evasion:

- $K_L \rightarrow \pi\pi$ implying CP requires Superposition Principle of QM -- give up SuPoPr!

- $\exists$ invisible CP odd particle $U$ \( K_L \rightarrow \pi^+\pi^- [U] \)
  
  a la Pauli’s postulate for $\nu$’s in $\beta$ decay $n \rightarrow p\ e\ [\nu]$  
  
  introduce new invisible particle to save conservation law

\[
\begin{array}{cc}
U & \nu \\
CP & \text{energy-momentum} \\
did not work & did work
\end{array}
\]

Homework #2: How was it ruled out experimentally?

`quod licet Jovi, non licet bovi’

\[
\begin{array}{cc}
= \text{Pauli} & = \text{non-Pauli}
\end{array}
\]
Findings of Fitch-Cronin exp. soon accepted since `perpetrators' were considered `real pros'.

frustrating -- no `peccate fortiter'
CP invariance as a `near miss' -- BR ~ 0.002 -- vs. maximal P

'64 - '72: lack of theory not realized --
even after renormalizibility of SU(2)\(_L\)xU(1) recognized
(except for short remark by Mohapatra in '72)

Wolfenstein's `Superweak Model' is not a theory,
not even a model -- it is merely a classification scheme.

Phenomenology quickly developed.
(3.1) Phenomenology of CP, Part 1

Phenomenological distinction between $\Delta F=1$ & $\Delta F=2$ dynamics; yet underlying theory has to yield both!

Same interplay between $\Delta F=1$ & $\Delta F=2$ affects CP

\[
\begin{align*}
K^0 & \overset{\Delta S=2}{\leftrightarrow} K^0 \\
K_S & \overset{\Delta S=1}{\rightarrow} \pi\pi \\
K_L & \overset{\Delta S=1}{\rightarrow} \pi\pi
\end{align*}
\]

\[
\eta_{+-,00} = \frac{A(K_L \rightarrow \pi^{+,0} \pi^{-,0})}{A(K_S \rightarrow \pi^{+,0} \pi^{-,0})}
\]

\[
\eta_{+-} = \epsilon + \epsilon', \quad \eta_{00} = \epsilon - 2\epsilon'
\]

As long as CP seen in a single decay of a neutral meson, distinction between direct & indirect CP somewhat arbitrary!
direct $CP$

the intervention of Penguins

- sensitive to 3 families in $\Delta S = 1$
- CKM not a superweak theory
  - $\epsilon'/\epsilon$ suppressed by
    - $\Delta I = 1/2$ rule
    - being a loop effect
    - superheavy top mass

\[
\frac{\epsilon'}{\epsilon} = \frac{\log m_t}{(m_t/M_W)^2}
\]
3 types of CP observables in oscillations

- existence
- asymmetry in initial state
- asymmetry in final state

\[ K_L \rightarrow \pi^+ \pi^- \]

\[ K^0 \rightarrow \pi^+ \pi^- \text{ vs. } \bar{K}^0 \rightarrow \pi^+ \pi^- \]

\[ K_L \rightarrow \ell^+ \nu \pi^- \text{ vs. } \ell^- \nu \pi^+ \]

Convention indep. definition of `+' & `-', matter & antimatter, `L' & `R'

NR Schroedinger eq.

\[
i \frac{d}{dt} \begin{pmatrix} |K^0> \\ |\bar{K}^0> \end{pmatrix} = \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12} - i\Gamma_{12}/2 & M_{11} - i\Gamma_{11}/2 \end{pmatrix} \begin{pmatrix} |K^0> \\ |\bar{K}^0> \end{pmatrix}
\]
Diagonalize $\mathcal{M} - i\Gamma/2$

- **eigenvalues** $M_{L,S} \& \Gamma_{L,S}$ of mass $ES$
- **coefficients** $q/p$ for
  
  $K_L \propto p|K^0> + q|\bar{K}^0>$, $K_S \propto p|K^0> - q|\bar{K}^0>$

  $$q/p = \left[ \frac{(M^{*}_{12} - i\Gamma^{*}_{12}/2)/(M_{12} - i\Gamma_{12}/2)}{M_{12}} \right]^{1/2}$$

  $M_{12} = Re \langle K^0|L_{eff}(\Delta B=2)|K^0 \rangle$

  The phase of $q/p$ convention dependent

  $|q/p| \neq 1$ CP observable

Homework #3: $\langle K^0|\bar{K}^0 \rangle = 0$ -- yet $\langle K_L|K_S \rangle = ?$

Single real number $\Phi(\Delta S=2) = \arg(M_{12}/\Gamma_{12})$ controls 3 types of CP observables
\[ \text{dominant for } \Delta M(K) \]

\[ \text{dominant for } \Delta M(B) \]

\[ \text{Homework #4: } \Delta M(B) \propto (m_t/M_w)^2 \to \infty \text{ as } m_t \to \infty \text{ -- no de-coupling ?!} \]
'86: discovery of $B^0$ oscillations: $\chi(B_d) = \frac{\Delta M(B_d)}{\Gamma_B} = 0.75$
  ➡ indirect bound $m_t > 100 \text{ GeV}$
  [similar, though less precise than later LEP I findings]
- optimal, since
  oscillation rate $\Delta M(B_d) \sim$ decay rate $\Gamma_B$

theorem:

Consider a neutral meson $P^0$ decaying into a final state $f$ that is a CP ES. If the decay rate evolution in (proper) time is not described by a single exponential, then CP!

Homework #5: Prove it!
due to CPT CP through complex phases
→ need 2 different, yet coherent amplitudes

Observables?

Caveat: must be re-phasing invariant under $|B^0\rangle \rightarrow e^{-i\xi}|B^0\rangle$

- $|T(B \rightarrow f)| \neq |T(B \rightarrow f)| \quad \Delta B = 1$
- $|q| \neq |p| \quad \Delta B = 2$
- $\text{Im}(q/p)\rho(f)$, $\rho(f)=T(B\rightarrow f)/T(B\rightarrow f) \quad \Delta B = 1 & \Delta B = 1$

- $B^{ch}, \Lambda_b$: only direct CP
- $B^0 \rightarrow l^+ X$ vs. $B^0 \rightarrow l^- X$: only indirect CP within SM
- $B^0/B^0 \rightarrow f$: direct & indirect CP
(3.2.1) Direct CP without Oscillations

\[ \Gamma(B^- \rightarrow f^-) - \Gamma(B^+ \rightarrow f^+) \propto \sin(\phi_1^w - \phi_2^w) \sin(\delta_1^{str} - \delta_2^{str}) \]

→ need both nontrivial weak & strong phase shifts!

1987: \( BR(B_d \rightarrow \pi^- K^+) \sim 10^{-5} \), \( A_{CP} = -0.10 \)
(3.2.2) CP involving Oscillations

\( B_d \rightarrow \psi K_S \)

\( \Delta B = 2 \)

\( \Delta B = 1 \)

\( \psi K_S \)

\( t_{\text{dec}} \)

\( \text{direct } CP \)

\( \text{indirect } CP \)

\( t = 0: \quad B_d \rightarrow \psi K_S \)

\( \Delta B = 1 \)

\( \psi K_S \)

\( t_{\text{inter}} \)

\( B_d \rightarrow \psi K_S \)

\( \text{rate}(B_d \rightarrow \psi K_S) \propto e^{-\Gamma t}(1 - [+] A \sin \Delta m_d t) \)
\[ A = \text{Im}(q/p)\bar{\rho}(f), \quad \bar{\rho}(f) = T(B \to f)/T(B \to f) \]

For \( f = B_d \to \psi K_S \) we have

\[ A(B_d \to \psi K_S) = \sin 2\phi_1 \]

with no hadronic uncertainty!

\[
\sin 2\phi_1 = \begin{cases} 
\text{up to unity} & \text{our paper (1980)} \\
0.6 - 0.7 & \text{from } \epsilon_K/\Delta M(B) \text{ [=}f(m_t)] \text{ (1993, i.e. before discovery of top)} \\
0.72 \pm 0.07 & \text{from CKM fits in 1998}
\end{cases}
\]
3.3 The Completion of a Heroic Era

direct CP established by '99

- WA '03: \[ \text{Re } \varepsilon' / \varepsilon = (1.66 \pm 0.16) \times 10^{-3} \]
  \[\frac{\Gamma(K^0 \rightarrow \pi^+\pi^-) - \Gamma(K^0 \rightarrow \pi^+\pi^-)}{\Gamma(K^0 \rightarrow \pi^+\pi^-) + \Gamma(K^0 \rightarrow \pi^+\pi^-)} = (5.5 \pm 0.6) \times 10^{-6}\]

- a discovery of the first rank -- irrespective of theory

- experimental groups earned our admiration

- not inconsistent with SM/CKM
  
  do not expect quick conclusive reply from theory
`Never underestimate Nature’s ability to come up with an unexpected trick!’

Physicists had thought to have seen it all after the discovery of $P$ -- and then the earthquake of $CP$ struck.

* CKM Theory -- `all it does, it works’.

A theory based on a set of mass related basic quantities -- fermion masses, CKM parameters -- that any sober person would view as frivolous -- were they not forced upon us by data -- in particular since we have no deeper understanding of mass generation in particular for fermions

* The SM has to produce a host of large $CP$ in $B$ decays -- there is no plausible deniability